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A FILM MODEL FOR THE PREDICTION OF FLOODING AND FLOW REVERSAL FOR GAS-LIQUID FLOW IN VERTICAL TUBES

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Abstract—A model is presented which demonstrates that the process of flooding and flow reversal can be explained on the basis of a film mechanism. The model predicts well the gas flow rate at which flooding and flow reversal begins and ends for a given liquid flow rate and the presence of a hysteresis loop between flooding and flow reversal. The predictions of the theory are in satisfactory agreement with experimental flooding data.

INTRODUCTION

The process of flooding in vertical pipes or channels is an interesting and important phenomenon that has attracted the attention of many investigators. It occurs for countercurrent flow of gas and liquid where the liquid flows downward and the gas upward. Consider a pipe where liquid is introduced at the middle as a film, while gas flows from the bottom to the top (see figure 1). For low or zero gas flow rates the liquid will flow downward. As the gas rate increases, some of the liquid will be carried upward so that simultaneous upward and downward liquid film flow will exist. Finally, for sufficiently high gas flow rates the liquid will flow only in the upward direction. Conversely, if the gas flow rate is kept constant and one increases the liquid feed rate, one finds that at low liquid rates and moderate gas rates the liquid will flow down. With increasing liquid feed a maximum value for downward flow in the film is reached. Increasing the liquid feed rate further will not alter this maximum downflow rate, but the "excess" liquid will move upward. Thus, for each gas flow rate there exists a maximum value of the liquid film flow rate that can move downward countercurrent to the gas flow. This downward flow rate of liquid cannot be increased unless the gas flow rate is decreased. Thus, the flooding phenomenon is a process that limits the downward countercurrent liquid flow rate.

The flow reversal phenomenon is the inverse of the flooding process. At high gas flow rates a climbing film exists. As the gas flow rate is decreased a point will be reached where partial downflow of liquid takes place. Further decrease in the gas flow rate will result in dryout of the climbing film and the flow returns to the condition of a downward falling film with upflow of gas.

Flooding experiments have been carried out by many investigators in basically two types of experimental equipment. In the first type the liquid is introduced in a mid-section of the pipe through a well designed entrance device, normally through a sintered tube, that insures homogeneous distribution of the liquid around the inner periphery of the pipe. Equipment of this type was used by Hewitt & Wallis (1963), Hewitt *et al.* (1965), Clift *et al.* (1966), Suzuki & Ueda (1977) and Dukler & Smith (1977).

A second experimental arrangement consists of upper and lower tanks with the liquid entering the pipe from the top tank by gravity. This arrangement was used by Imura *et al.* (1977) and Bharathan, *et al.* (1978). Pushkina & Sorokin (1969) used both types. Although experiments with feed located at the top are easier to carry out, this arrangement makes it impossible to explore the phenomena of transition from downflow to upflow in the film. Furthermore, the flow reversal process cannot be explored.

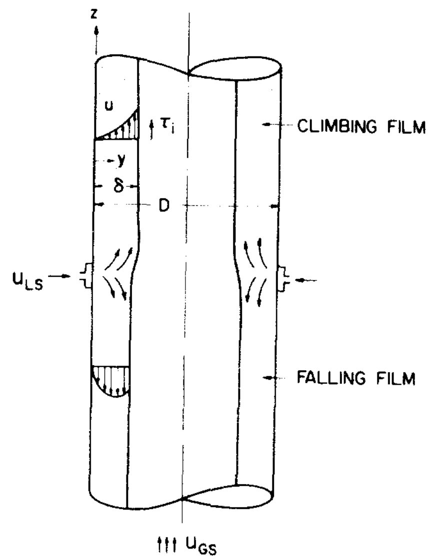

Figure 1. Climbing and falling film.

A review of experiments, correlations and analytical approaches to flooding which have appeared in the published literature and reports has been presented by Dukler & Smith (1977) and Tien *et al.* (1979). A large variety of empirical correlations have been proposed, the most widely used being that of Hewitt & Wallis (1963).

$$\left\{\frac{U_{GS}\rho_G^{1/2}}{[gD(\rho_L-\rho_G)]^{1/2}}\right\}^{1/2}+\left\{\frac{U_{LS}\rho_L^{1/2}}{[gD(\rho_L-\rho_G)]^{1/2}}\right\}^{1/2}=C. \qquad [1]$$

The constant, C, has been shown to vary with the equipment size, the range of flow velocities and in some cases the entry configuration and fluid properties. In this equation U_{GS} and U_{LS} are the superficial velocities of the gas and liquid respectively, ρ_G and ρ_L the gas and liquid densities, g the acceleration of gravity and D the tube diameter. Many modifications of this equation have been proposed based on specific data sets.

Approaches to a theoretical prediction of the flood point based on physical mechanism have been few in number. Shearer & Davidson (1965), Centinbudakla & Jameson (1969), as well as Imura *et al.* (1977) all suggest that flooding takes place as the result of the formation of an unstable wave that rapidly grows until it bridges the tube. Then the liquid is carried up by the gas as a slug or as an entrained phase. In this case the prediction of the flood point comes down to predicting the condition for instability. Measurements by Dukler & Smith (1977) and Suzuki & Ueda (1977) have convincingly shown that bridging does not occur. Furthermore, none of these theories show agreement with experiment except over narrow ranges of data, usually for small tube diameters and at low pressures.

Dukler & Smith (1977) observed the presence of drops at or just below the gas rate at which flooding takes place. They suggested that flooding takes place when the gas rate is sufficient to lift the largest drop against gravity. A mathematical model of this process results in the following theoretical equation for the flooding condition

$$\frac{U_{GS}\rho_G^{1/4}}{[\sigma g(\rho_L-\rho_G)]^{1/4}}=2.8\,. \qquad [2]$$

Pushkin & Sorokin (1969) developed an identical result using experimental flooding data and dimensional analyses and obtained the constant 3.2 as against 2.8 above. The approach above gives a theoretical basis for the formerly empirical result. However, the model predicts unrealistically large drop sizes under certain conditions and is in poor agreement with some published data.

In this paper a relatively simple, self-consistent film model is proposed that appears to explain the process of flooding and flow reversal. It ignores the presence of waves and attributes the process of flooding to the balance between gravity and interfacial shear on the smooth film.

THEORY

Figure 1 shows the geometry of film flow in the vertical pipe. Liquid is introduced circumferentially somewhere between the top and bottom of the pipe where gas flows upwards. At gas rates below flooding the liquid flows downward as a falling film. At the start of flooding a small part of the liquid flows upward as a film and a situation of simultaneous falling and climbing films exists as shown in figure 1. As the gas rate is increased the fraction of feed liquid which flows upward increases until a gas rate is reached where all of the liquid flow is upward in the film.

A force balance for steady laminar flow on a thin annular ring of film extending from the wall a distance y into the liquid gives

$$\tau = \mu_L \frac{\mathrm{d}u}{\mathrm{d}y} = \tau_i - \left(\frac{\mathrm{d}p}{\mathrm{d}z} + \rho_L g\right)\left(\delta - y\right) \qquad [3]$$

where τ is the shear stress at y, τ_i the interfacial stress, u is the local liquid axial velocity, z and y the axial and radial coordinates, δ the film thickness, μ_L and ρ_L the liquid viscosity and density, p the static pressure and g the acceleration of gravity.

Equation [3] assumes thin films, that is $\delta \ll D/2$ where D is the tube diameter. The comparable equation for thick films has been presented (see Hewitt & Hall Taylor 1970); however, numerical computations show little difference in the results. Thus, the simpler form above has been used in the development which follows.

Solving [3] for velocity

$$u = \frac{1}{\mu_L}\left[\tau_i y - \left(\frac{\mathrm{d}p}{\mathrm{d}z} + \rho_L g\right)\left(\delta y - \frac{y^2}{2}\right)\right]. \qquad [4]$$

Note that u is designated positive in the positive (upward) z direction. Integrating [4] provides the liquid flow rate in the film in terms of the superficial liquid velocity, U_{LS}. This is negative for net downflow and positive for net upflow.

$$U_{LS} = \frac{4}{\mu_L D}\left[\frac{1}{2}\tau_i \delta^2 - \frac{1}{3}\left(\frac{\mathrm{d}p}{\mathrm{d}z} + \rho_L g\right)\delta^3\right]. \qquad [5]$$

A force balance on the gas core yields

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\frac{4\tau_i}{D - 2\delta} - \rho_G g \qquad [6]$$

where ρ_G is the gas density. Substituting in [5] and rearranging, yields

$$\tau_i = \frac{U_{LS}\mu_L D}{2\delta^2} + \frac{2}{3}(\rho_L - \rho_G)g\delta. \qquad [7]$$

Now it is necessary to relate τ_i to the gas flow rate. The basic mechanisms which determine interfacial shear are poorly understood and the relationships which are available are largely empirical. It is not the purpose of the work reported here to address that problem. Instead, we adopt one of the correlations recently suggested which does an adequate job of describing τ_i. One approach correlates the friction factor in terms of the relative film thickness δ/D. Wallis *et al.* have recommended several such correlations, the most recent of which (Bharathan 1978) proposes the form,

$$f_i = A + B\left(\frac{\delta}{D}\right)^n. \tag{8}$$

Thus, the equation

$$\tau_i = \frac{1}{2} f_i \rho_G \frac{U_{GS}^2}{(1-2\delta/D)^4} \tag{9}$$

yields the required relation between the gas flow rate U_{GS}, the liquid flow rate U_{LS} and the film thickness δ. In dimensionless form, this relation takes the form

$$U'_{GS} = \left\{\frac{2(1-2\delta')^4}{A+B\delta'^n}\left[\frac{U'_{LS}}{2\delta'^2} + \frac{2}{3}\delta'\right]\right\}^{1/2} \tag{10}$$

where

$$\delta' = \frac{\delta}{D} \tag{11}$$

$$U'_{LS} = \frac{\mu_L}{D^2(\rho_L - \rho_G)g} U_{LS} \tag{12}$$

$$U'_{GS} = \frac{\rho_G^{1/2}}{(\rho_L - \rho_G)^{1/2}\sqrt{(gD)}} U_{GS}. \tag{13}$$

Given U'_{LS} and δ' one can calculate U'_{GS}. Note that U'_{LS} is negative for falling films and positive for upflow. In order to obtain a solution to equation [10] the coefficients A and B and the exponent n which define f_i must be known. Bharathan (1978) recommended the following values: For a 2.5 cm pipe, $A = 0.005$, $B = 280$ and $n = 2.13$. For a 5.1 cm pipe, $A = 0.005$, $B = 406$ and $n = 2.04$. Clearly, before this approach can provide a general solution to predicting flooding, it will be necessary to have a sound non-empirical basis for predicting the interfacial friction factor.

The solutions given by [10] appear in figures 2 and 3 for pipe diameters of 2.5 and 5.1 cm respectively. The results are conveniently represented as curves of U'_{GS} vs δ/D for parameter of U'_{LS}. Negative U'_{LS} indicate downflow. It is important to note that in general once U'_{LS} and U'_{GS} are specified, three formal solutions exist. That is, the force balance is satisfied for each flow rate pair downflow at two film thicknesses and for upflow at one. However, the results also show that above a certain value of U'_{GS} no solution for downflow exists and we presume this to indicate the end of the flooding process and the existence only of upflow. On the basis of the analysis which is presented below we conclude that the dashed portions of each curve represent unstable physical conditions. As a result it seems a reasonable physical speculation that curves connecting the maximum values of U'_{GS} for stable downflow represent the locus of points along the flooding curve. In a similar manner a curve connecting the minimum values of U'_{GS} for stable upflow represents the locus points along the flow reversal curve.

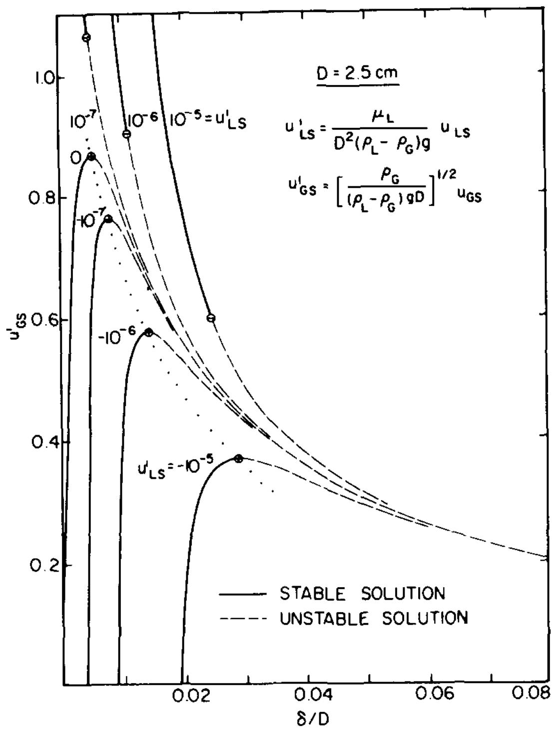

Figure 2. Film thickness for falling and climbing films in 2.5-cm dia. pipe.

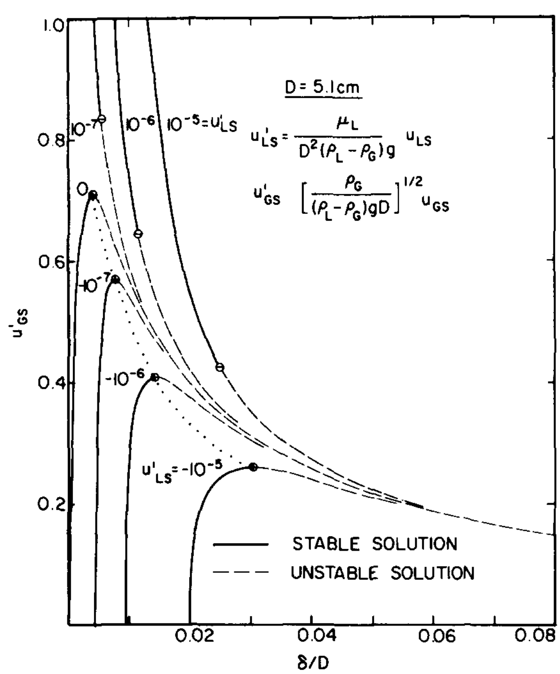

Figure 3. Film thickness for falling and climbing films in 5.1-cm dia. pipe.

When multiple solutions exist from the formulation of a physical problem, one searches for criteria on which to decide which solutions are physically realistic and thus can be expected to be observed. Two criteria are applied here: (a) stability analysis and (b) limiting flow as dictated by kinematic wave theory.

Stability analysis

Define the dimensionless interfacial shear as

$$\tau_i' = \frac{\tau_i}{(\rho_L - \rho_G)gD}. \quad [14]$$

Equation [7] which results from a force balance on the liquid, becomes

$$\tau_i' = \frac{U_{LS}'}{2\delta'^2} + \frac{2}{3}\delta'. \quad [15]$$

The relation between τ_i' and δ' as dictated by this equation is shown as the solid lines in figure 4 for parameter values of U_{LS}'. An independent expression for τ_i is obtained from gas phase through [8] and [9].

$$\tau_i' = [A + B(\delta')^n]\frac{(U_{GS}')^2}{2(1-2\delta')^4}. \quad [16]$$

The family of broken curves gives the relationship between τ_i' and δ' as dictated by this equations for a 2.5-cm dia. tube. The intersection of any two curves gives the solution for that particular flow rate pair. For example, consider an absolute value of $U_{LS}' = 10^{-6}$ and $U_{GS}' = 0.5$. Three solutions exist, indicated by points A and B for downflow where $U_{LS}' = -10^{-6}$ and C for upflow where $U_{LS}' = +10^{-6}$.

The stability of these solutions can be determined by the manner used when multiple solutions are encountered in a continuous stirred tank (CSTR) where a chemical reaction takes place. In that case a graph of concentration vs temperature is used and the steady state of a CSTR is given by the points of intersection of the material and energy balance curves. In this case τ' and δ' are analogous to the concentration and temperature in a CSTR.

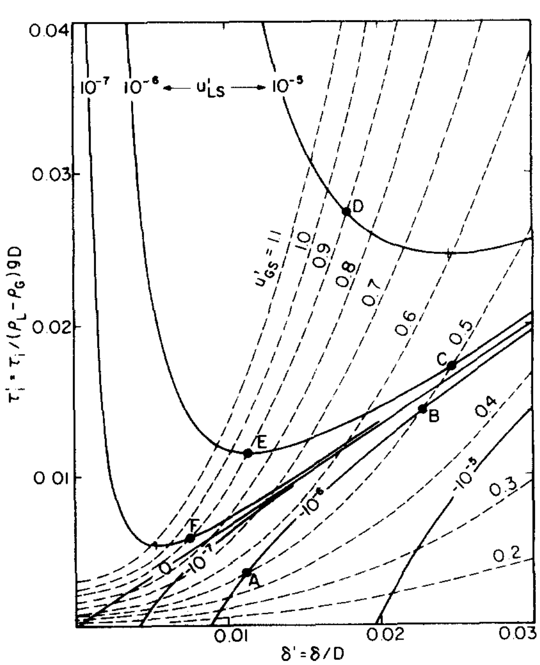

Figure 4. Stability diagram D = 2.5 cm.

Consider the situation at point A. If δ' randomly increases by a small amount, one can observe that the shear stress provided by the gas (the broken line) is less than that needed to maintain this new film thickness. As a result the film thickness will decrease and return to its original value at A. The solution at A is thus stable. Now consider the solution expressed by point B. If the film thickness momentarily increases due to a disturbance, the shear stress increase in the gas phase exceeds that needed by the liquid to maintain that thickness. Thus, the film thickness will increase still further and the condition will diverge from its original state. It is thus unstable and unlikely to be observed physically except over transient periods. For the flow rate pairs used for illustration the same conclusion applies to point C. An examination of these curves shows that of the two solutions for downflow ($U'_{LS} < 0$) only the one with the smaller film thickness is stable. In the case of upflow ($U'_{LS} > 0$) the τ'_i vs δ' curves represented by the solid lines display minima. Intersections which take place to the left of the minimum are stable, as at point D. Those to the right are unstable such as at F. Point E represents an intersection at the minimum which is at the condition of neutral stability. Experimental data for upward film flow confirm the existence of the minimum in the τ_i vs δ curves as shown here. Furthermore, experiments show that as the gas rate is decreased below the value for minimum shear, large disturbance waves appear (Hewitt & Hall Taylor 1970). This analysis suggests that the point is identically the condition where the film becomes unstable.

One may visualize the behavior of the system as follows: for zero gas velocity and a given value of U'_{LS} the liquid film flows downward and its thickness, δ', is given by the Nusselt solution ([10] for $U'_{GS} = 0$). As the gas rate increases, the film thickness increases as given by the single valued stabled solution. Finally, the maximum is reached and no solution exists beyond this point as U'_{GS} is increased. At this point, some of the liquid starts to move upwards and this is the initial point of flooding.

As the gas flow rate increases, the liquid flowing downward decreases along the locus of the maxima (the dotted line in figures 2 and 3), each maximum corresponds to a different downward liquid flow rate. One reaches the maximum of the maxima at $U_{LS} = 0$. This point is the end of the flooding process and thereafter the flow is only upward. Increasing the gas flow rate causes further decrease in the thickness of the climbing film.

In the flow reversal process the gas flow rate decreases and a point is reached at which the solution becomes unstable. For small liquid flow rates the film will become unstable above the gas velocity for $U'_{LS} = 0$. Then the flow reversal will follow the flooding curve in the reverse direction. For higher liquid flow rates the instability point occurs below the maximum of the curve $U'_{LS} = 0$. Therefore, upward film flow will persist even at gas velocities where downflow is possible. This explains the hysteresis observed in experiments. When the point of instability is reached a switch to the previous flooding path must take place. This result is consistent with experiment which shows little or no hysteresis at low liquid rates but a substantial hysteresis loop at high rates.

Kinematic wave analysis

Lighthill & Whitham (1955) presented a theory of kinematic waves, based on the continuity equations, which they used to analyze flood waves and traffic on highways. Once a local concentration was defined they were able to show that flow limitations could be expected to take place when the velocity of propagation of a kinematic wave became zero. Zuber (1964) applied this idea to a series of problems in dispersed two phase flow. The liquid film thickness (or void fraction) can be considered a "concentration". Now it is of interest to explore the condition under which the velocity of propagation of a kinematic wave will be identically zero. Under these conditions small disturbances or changes in film thickness cannot propagate along the tube and flooding will result.

Define α as the tube section average void fraction. The equation of continuity written for the gas over a differential length of tube (gas density constant) is

$$\frac{\partial \alpha}{\partial t}+\frac{\partial}{\partial z}(\alpha U_G)=0 \qquad [17]$$

where t is time and U_G is the true average gas velocity. This equation can be rearranged as

$$\frac{\partial \alpha}{\partial t}+\left[\frac{\partial}{\partial \alpha}(\alpha U_G)\right]\frac{\partial \alpha}{\partial z}=0 \qquad [18]$$

$$\frac{\partial}{\partial \alpha}(\alpha U_G)=\frac{\mathrm{d}U_{GS}}{\mathrm{d}\alpha}=C_k. \qquad [19]$$

Equation [17] suggests that changes in the void fraction propagate in the z direction with a velocity, C_k. Thus to find the flooding point we search for the condition at which $C_k=0$. Since

$$\alpha=(1-2\delta')^2 \qquad [20]$$

C_k is zero when

$$\frac{\mathrm{d}U_{GS}}{\mathrm{d}\alpha}=\frac{\mathrm{d}U'_{GS}}{\mathrm{d}\delta'}=0. \qquad [21]$$

But the relationship between U'_{GS} and δ' has already been established in [10] and plotted in figures 2 and 3. This kinematic wave criteria thus dictates that flooding will take place at the location of the maxima for each parametric value of U'_{LS}. That this criteria is identical to the stability criteria discussed above is an interesting result, the significance of which is not yet apparent.

Discussion and comparison with theory

The film theory presented here is fully consistent with qualitative trends indicated in a variety of observation and experimental measurements.

(a) The gas rate required for flooding decreases with increasing liquid rate.

(b) The gas flow rate at the end of the flooding process is independent of the liquid flow rate.

(c) At low liquid rates the gas velocity required for flooding varies approximately as $D^{1/4}$.

(d) As the gas rate is increased the film thickness remains reasonably constant, until flooding is approached and then increases rapidly near the flood point.

(e) A hysteresis loop exists at higher liquid flow rates so that with decreasing gas rate downflow of liquid begins at lower gas flow rates than required to eliminate downflow as the gas rate is increased.

Quantitative comparisons between this theory and data appear in figures 5 and 6. Experimental data for flooding using air–water in a 5.1-cm tube appear in figure 5. The solid curve represents this theory and agreement with data is reasonably satisfactory. Prediction of [1], the empirical correlation of Hewitt & Wallis (1963), is shown as the dotted curve. Values of C varying from 0.8 to 1.0 have been suggested. The range of predictions between these two values is seen to bracket the data, however, no basis for selecting the correct value is apparent. The theoretical result for $D=2.5$ cm appears in figure 6 compared with Bharathan data for 2.5 cm dia. Agreement with data is seen to be good. Also included are the data of Hewitt *et al.* (1965) taken in a 3.2 cm dia. tube. This includes measurements where flow reversal information was

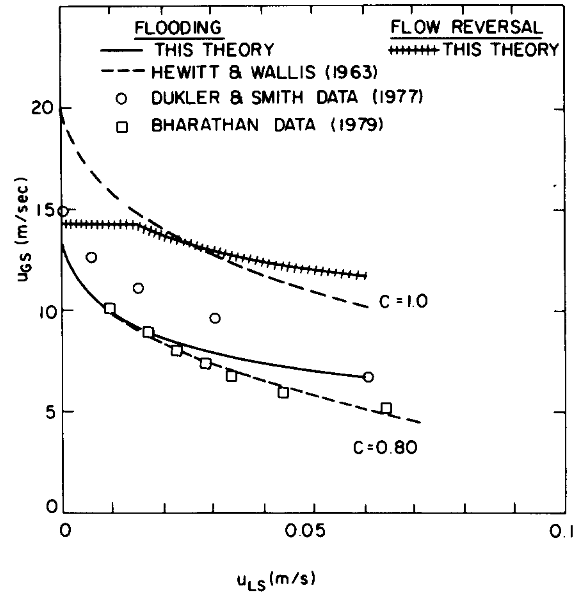

Figure 5. Comparison of theory and experiment D = 5.1 cm.

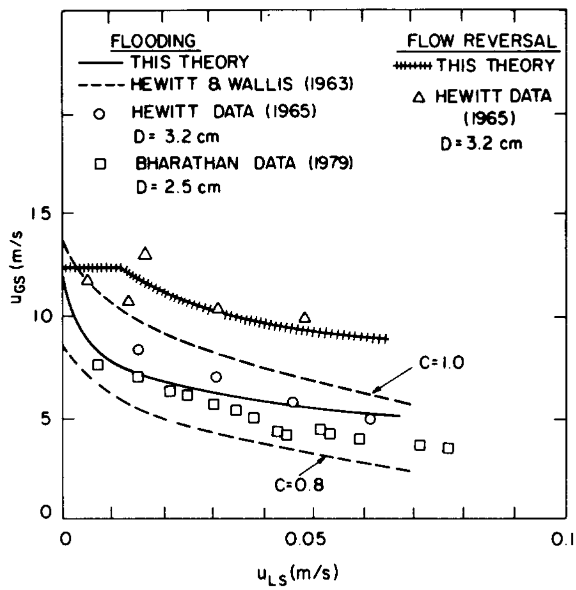

Figure 6. Comparison of theory and experiment D = 2.5 and 3.2 cm.

obtained. It was not possible to make theoretical calculations in the absence of interfacial friction factor data for their tube size. However, interpolation between the theoretical curves of figure 5 for 5.1 cm and figure 6 for 2.5 cm shares reasonably good agreement between this theory and data both for flooding and flow reversal. Note that the film Reynolds number for the range of data and calculations given in figures 5 and 6 is less than 1000 and thus the assumption of laminar flow is indeed valid.

Various studies have shown that entry configuration and length all influence the location of the flooding curve. This film theory suggests that such effects take place through the changes in interfacial shear which result from these different configurations and lengths. When comparisons are made between the predictions of this film theory which incorporates a realistic interfacial shear model for the experimental configuration actually used, the results are acceptable as indicated in figures 5 and 6. The deviations observed can readily be understood by the limitations of the friction factor correlations.

Summary and conclusions

A film model is developed which is applied to the prediction of the flooding and flow reversal process. The theory, which requires as an input an expression for the interfacial shear, predicts a variety of characteristics of the flooding process. The theoretical flooding and flow reversal curves are in satisfactory agreement with data. The influence of entry and exit configuration and tube length which have been reported to influence the flooding curve are suggested to take place through changes in interfacial shear.

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